M^2

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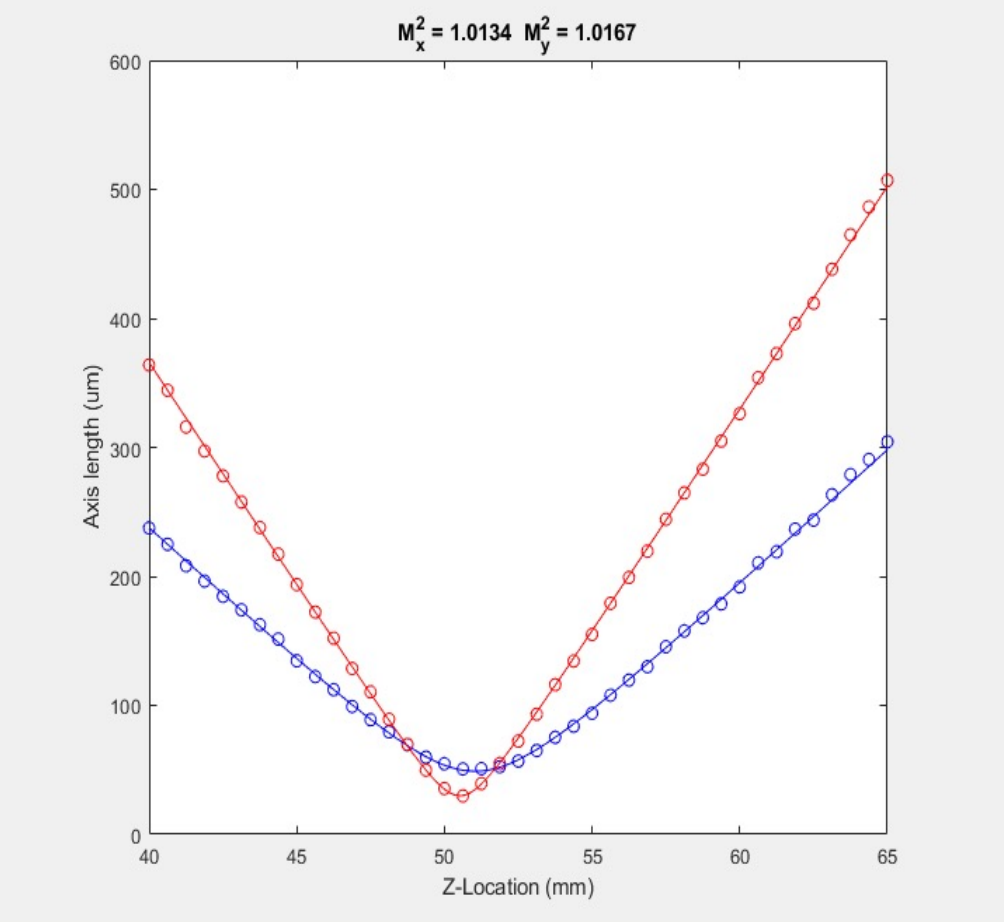
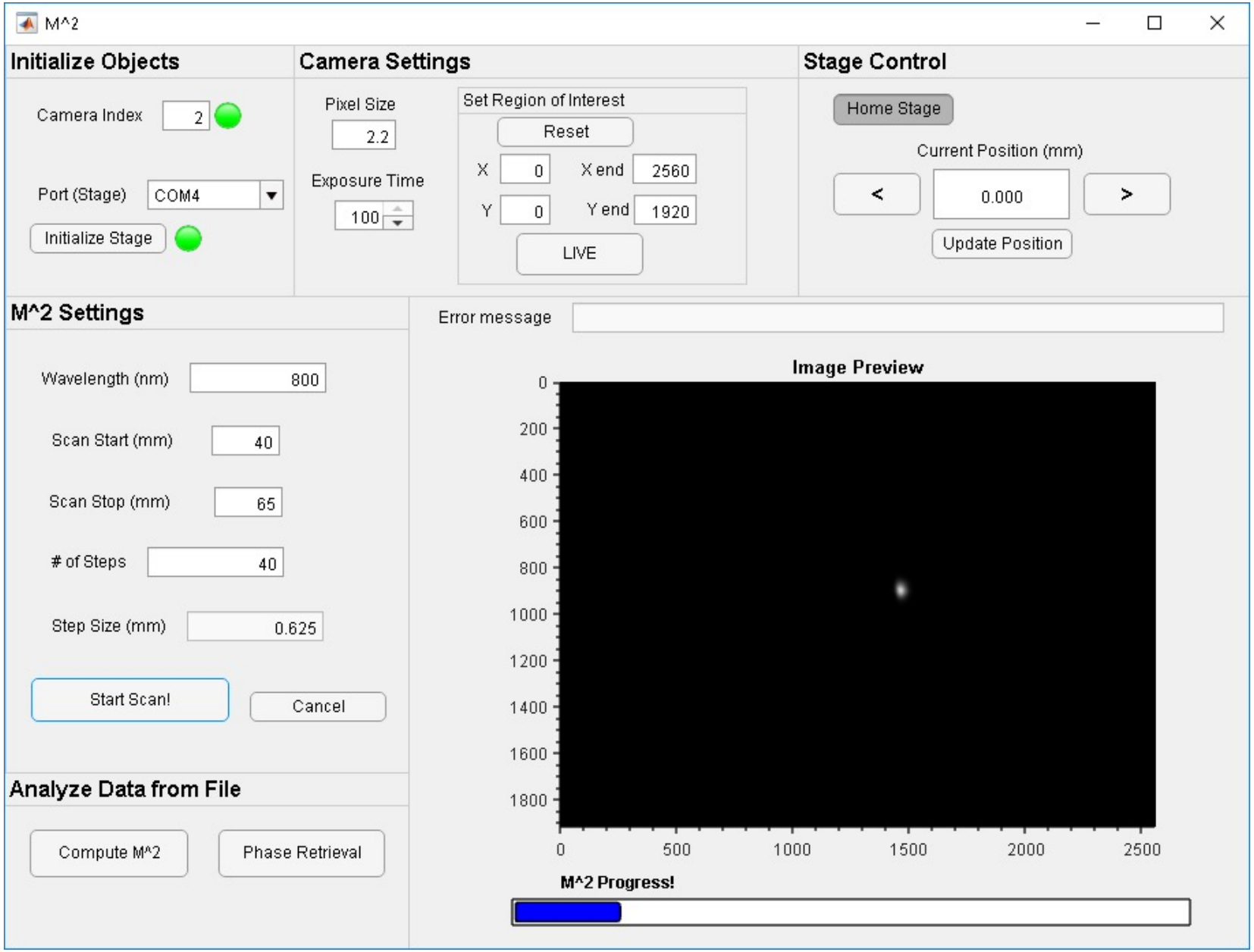
What is an M^2?

- M^2 is a measure of beam quality/astigmatism
- The M^2 factor indicates how well a beam can be focused.
- M^2 is sometimes called a times diffraction-limited factor
- A perfect diffraction-limited beam has an M^2 of 1 (for a Gaussian beam)



GUI


M^2
Initialize Objects
Camera Index
2
Port (Stage)
COM4
Initialize Stage
Camera Settings
Pixel Size
2.2
Exposure Time
100
Set Region of Interest
Reset
X
0
X end
2560
Y
0
Y end
1920
LIVE
Stage Control
Home Stage
Current Position (mm)
<
0.000
>
Update Position
M^2 Settings
Wavelength (nm)
800
Scan Start (mm)
40
Scan Stop (mm)
65
# of Steps
40
Step Size (mm)
0.625
Start Scan!
Cancel
Analyze Data from File
Compute M^2
Phase Retrieval
Error message
Image Preview
0
200
400
600
800
1000
1200
1400
1600
1800
0
500
1000
1500
2000
2500
M^2 Progress!

Calculating beam properties

- First-Order Moments

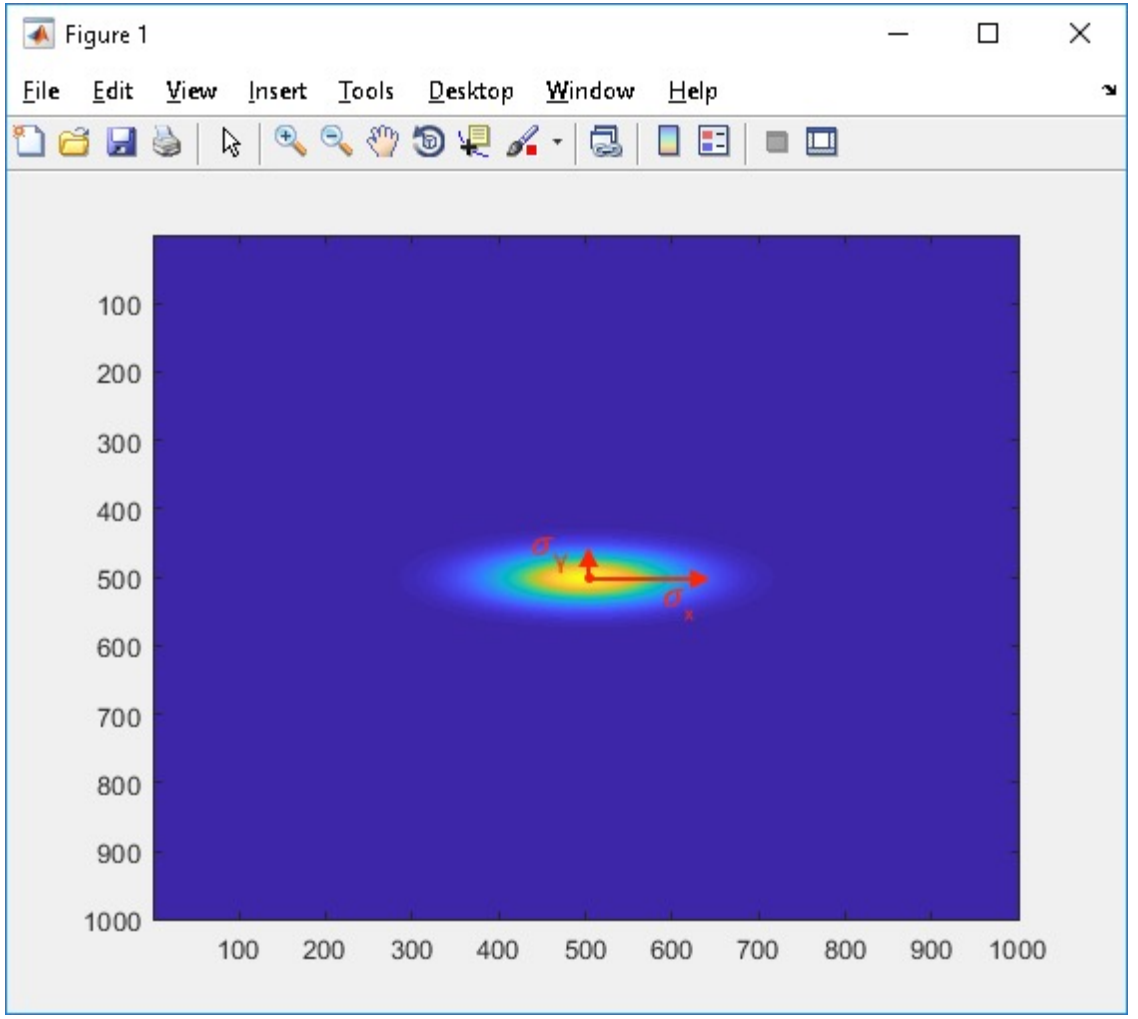
$$\overline{x} = \frac{\iint I(x,y)x\,dx\,dy}{\iint I(x,y)\,dx\,dy} \qquad \overline{y} = \frac{\iint I(x,y)y\,dx\,dy}{\iint I(x,y)\,dx\,dy}$$

- Second-Order Moments

$$\sigma_x^2 = \frac{\iint I(x,y)(x-\overline{x})^2\,dx\,dy}{\iint I(x,y)\,dx\,dy} \qquad \sigma_x = HW\frac{1}{\sqrt{e}}$$

$$\sigma_y^2 = \frac{\iint I(x,y)(y-\overline{y})^2\,dx\,dy}{\iint I(x,y)\,dx\,dy}$$

$$\sigma_{xy}^2 = \frac{\iint I(x,y)(x-\overline{x})(y-\overline{y})\,dx\,dy}{\iint I(x,y)\,dx\,dy}$$



Calculating beam properties

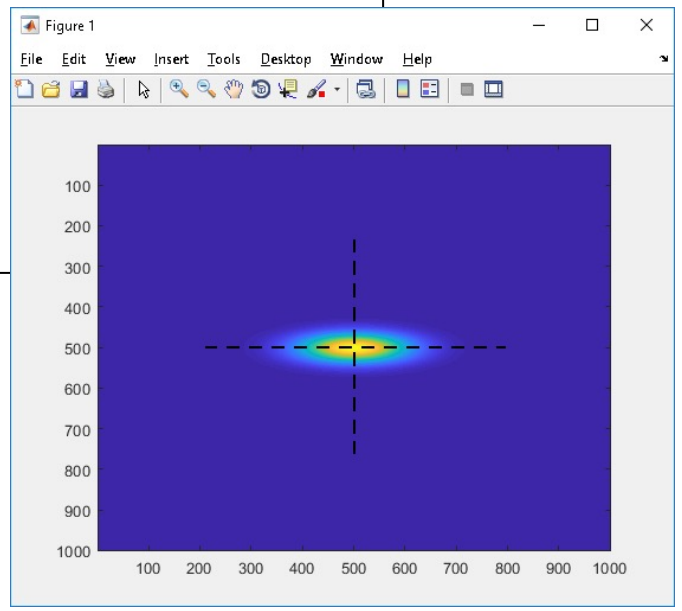
- Beam widths and the principle axes of the beam can be calculated from the second-order moments

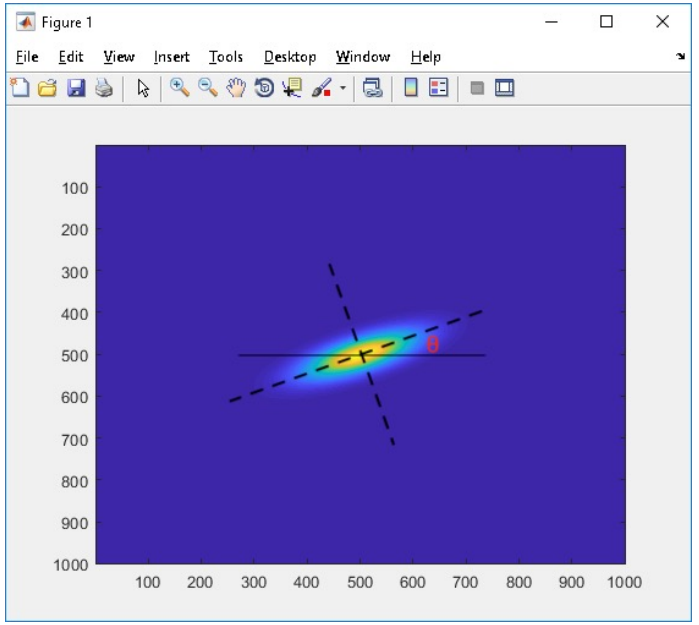
Beam widths:

$$d_{\sigma x}(z) = 2\sqrt{2}\left\{\left(\sigma_x^2+\sigma_y^2\right)+\gamma\left[\left(\sigma_x^2-\sigma_y^2\right)^2+4\left(\sigma_{xy}^2\right)^2\right]^{\frac{1}{2}}\right\}^{\frac{1}{2}}$$

$$d_{\sigma y}(z) = 2\sqrt{2}\left\{\left(\sigma_x^2+\sigma_y^2\right)-\gamma\left[\left(\sigma_x^2-\sigma_y^2\right)^2+4\left(\sigma_{xy}^2\right)^2\right]^{\frac{1}{2}}\right\}^{\frac{1}{2}}$$

$$d_{\sigma x} = FW\frac{1}{e^2}$$



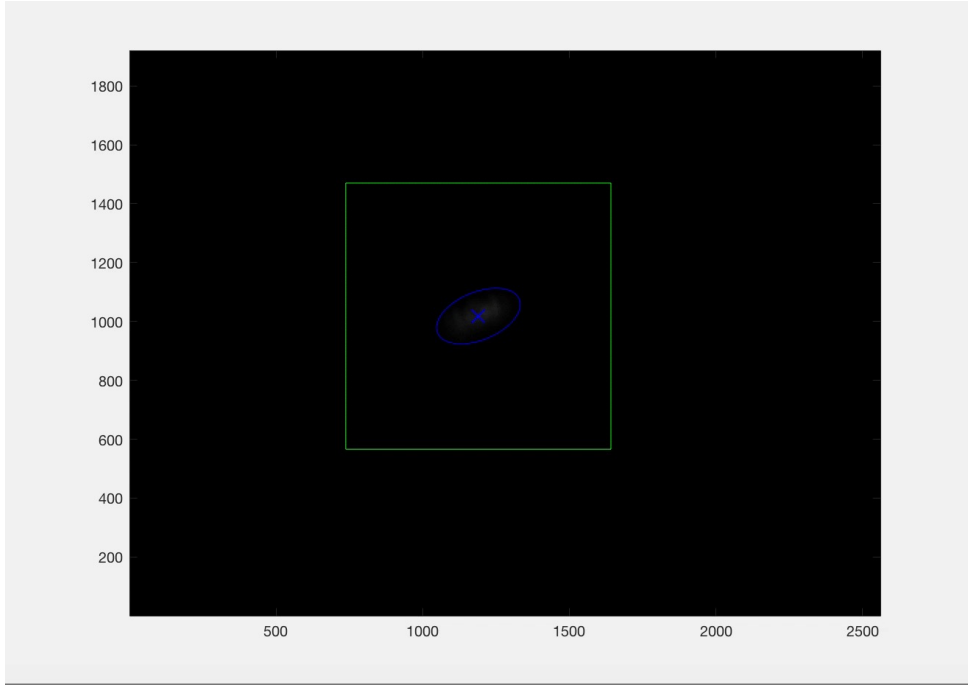


Angle:

$$\varphi(z) = \frac{1}{2}\arctan\left(\frac{2\sigma_{xy}^2}{\sigma_x^2-\sigma_y^2}\right) \qquad \theta = -\frac{\pi}{4} < \theta < \frac{\pi}{4}$$

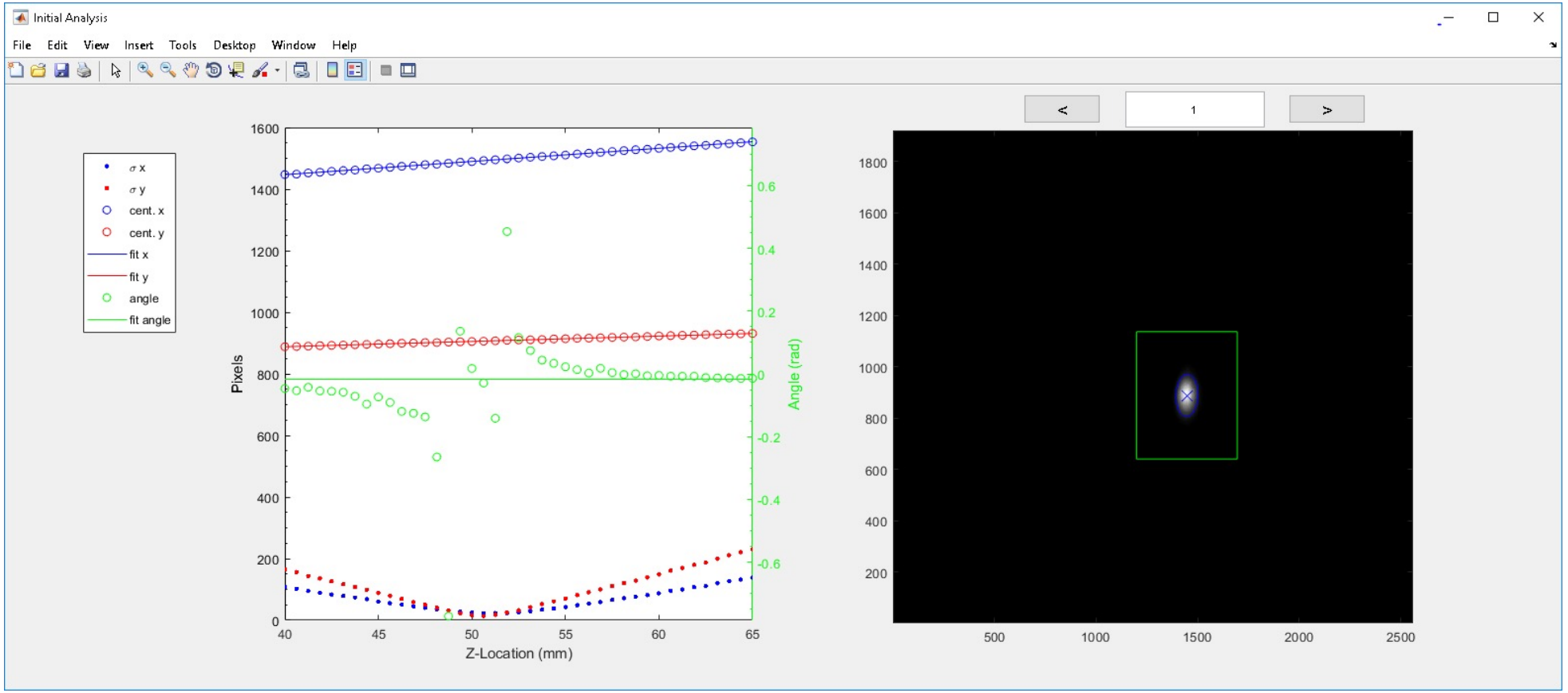
Finding a Region of Interest

- Calculation of σ_x^2 is sensitive to noise on edges
- Also, don't want to crop off beam intensity
- Iteratively find an integration box with size equal to $3d_{\sigma x}$ (or $3d_{\sigma y}$ depending on which is larger)
- Repeat for each image



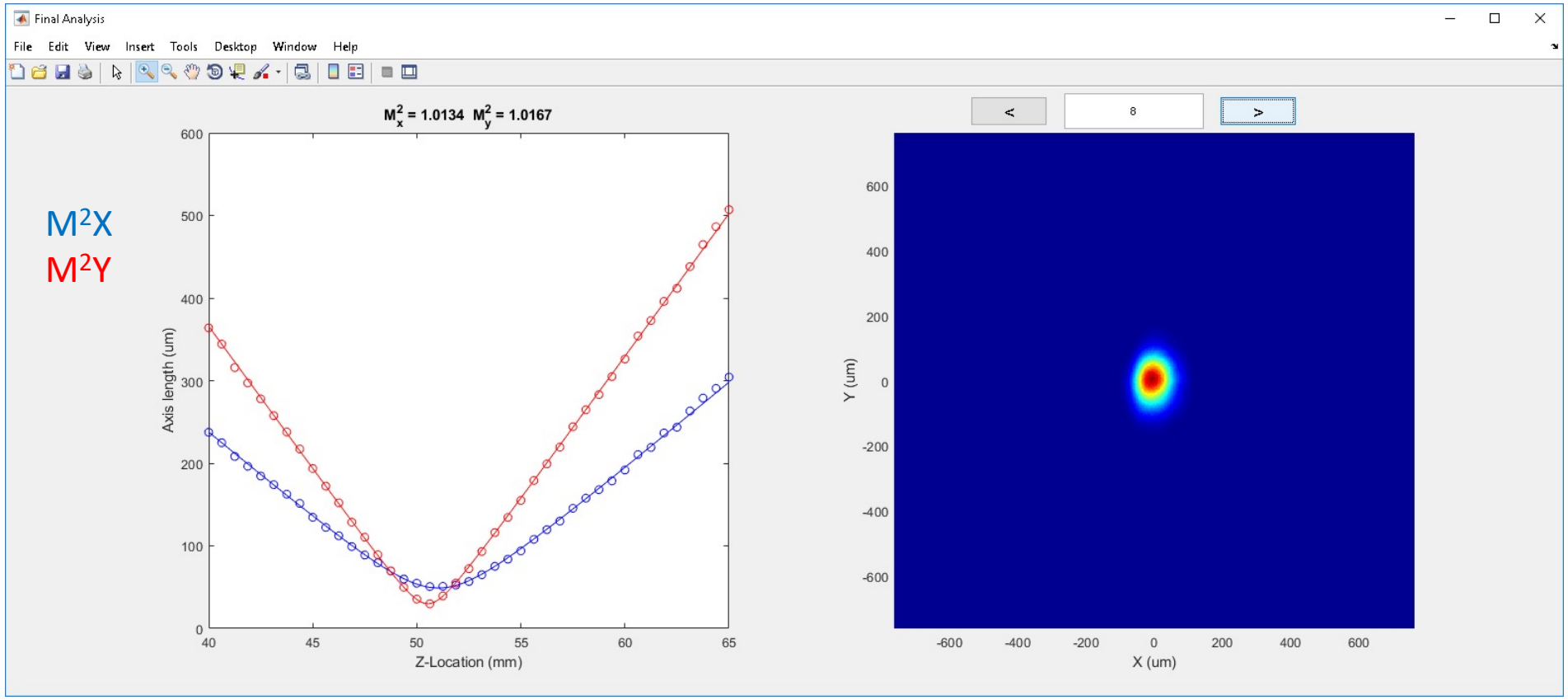
Initial Analysis

- Fit line to angle, because angle becomes unstable around beam waist, thus changing the definitions of $d_{\sigma x}$ and $d_{\sigma y}$ slightly for each image
- Fit line to centroids to account for possible error



Final Analysis

- Reanalyze images with fitted centroids and angle



Calculating M^2

- ISO Standard Equation returns complex values for some values of a, b, and c

$$d_\sigma(z) = \sqrt{a + bz + cz^2}$$

- Instead we used this equation, then converted to ISO Standard definitions of a, b, and c

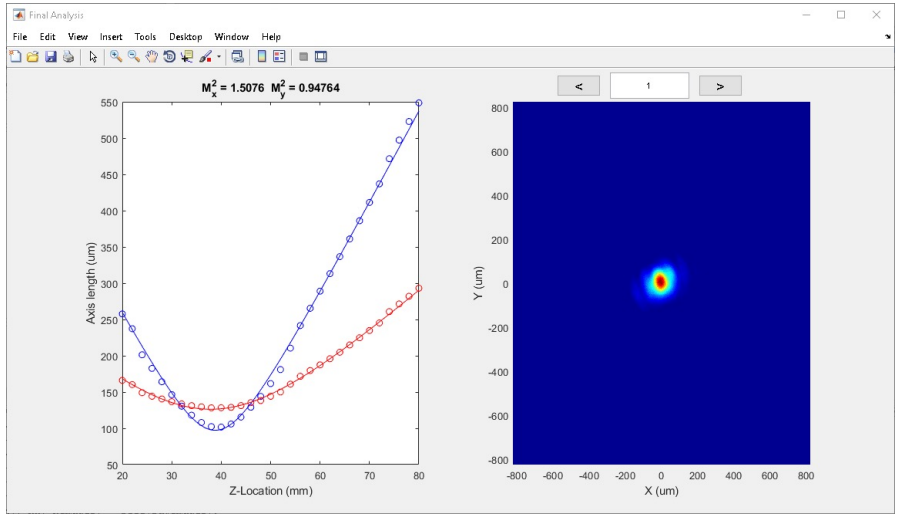
$$w_R(z) = w_{0R}\left[1 + \left(\frac{z\lambda M^2}{\pi w_{0R}^2}\right)^2\right]^{1/2}$$

Fit to:

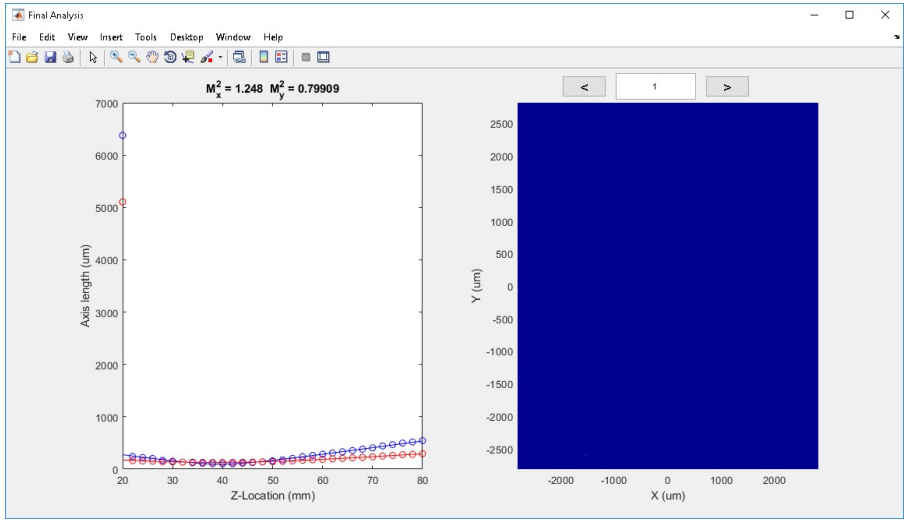
$$f(x) = \sqrt{a^2 + b^2(x-c)^2}$$

$$M^2 = \frac{\pi}{8\lambda}\sqrt{4ac - b^2}$$

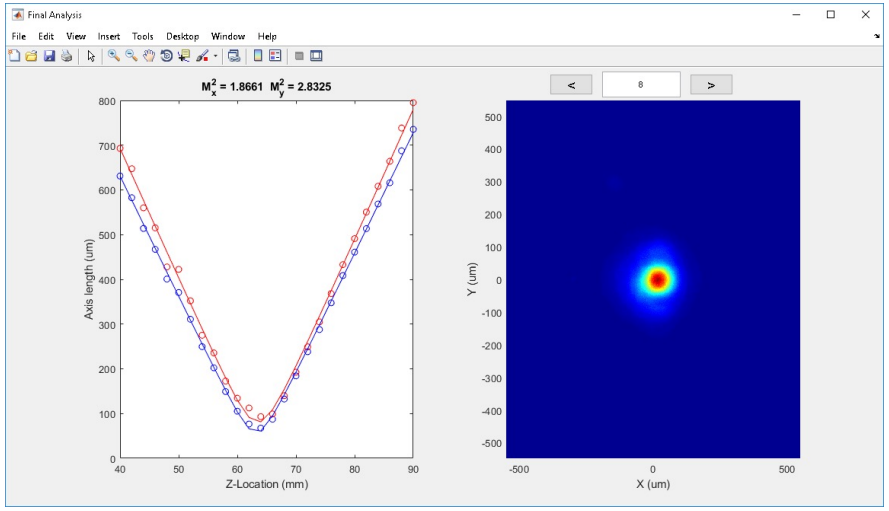
Sample Data



Diode Laser

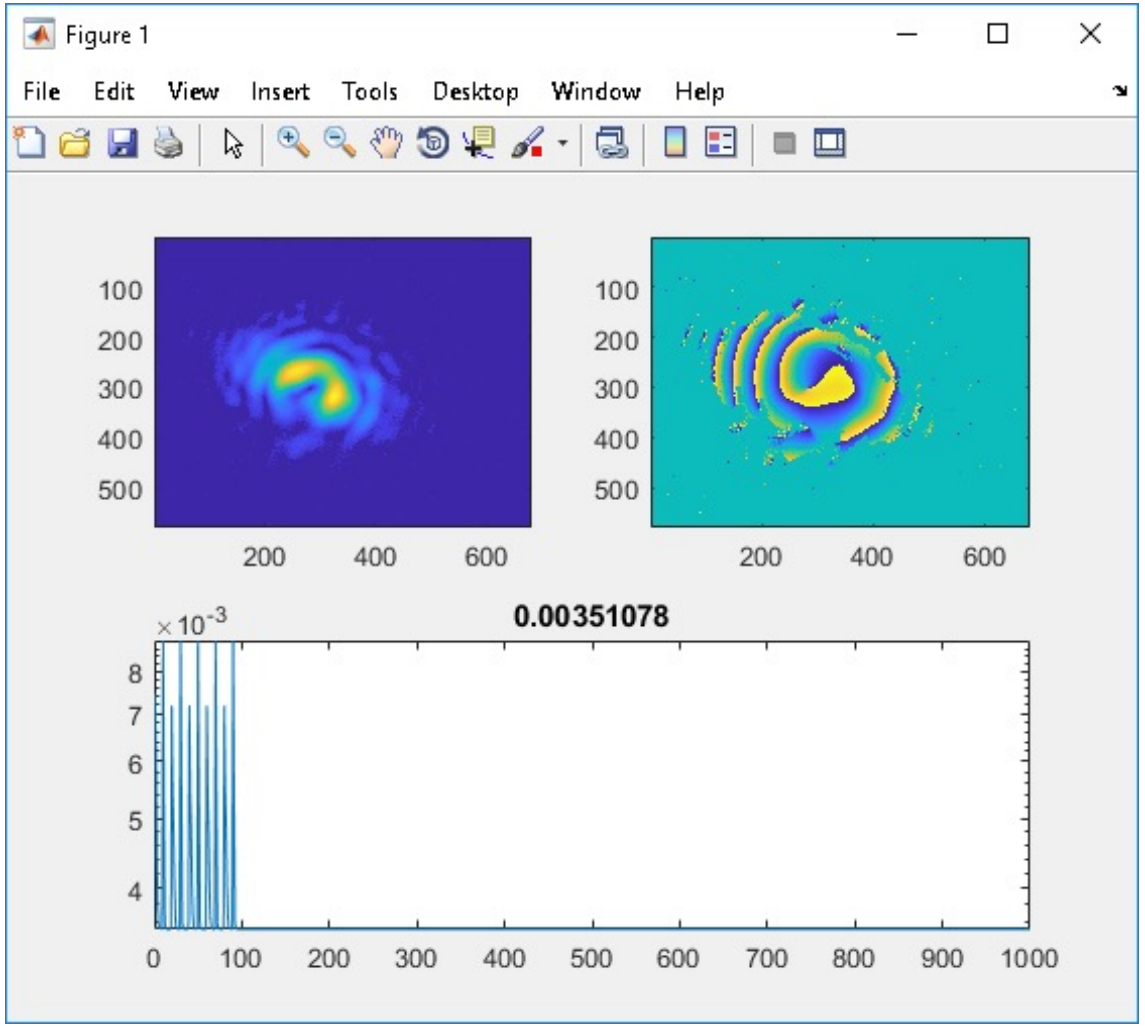


C-Wing Oscillator



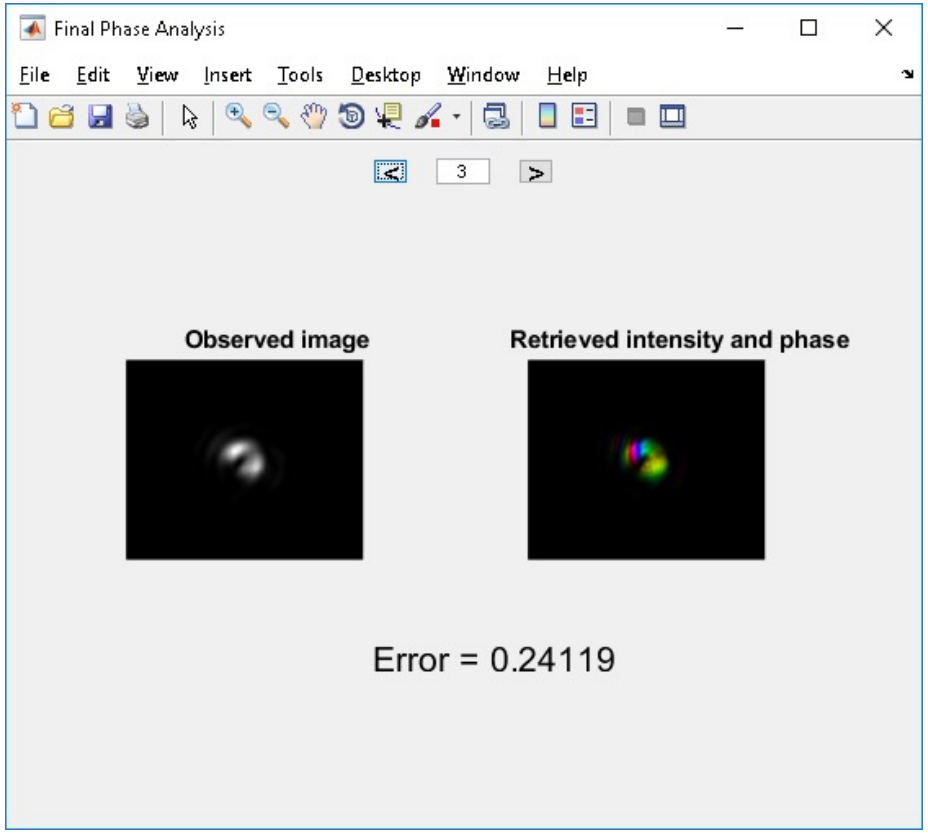
Phase Retrieval using Gerchberg-Saxton

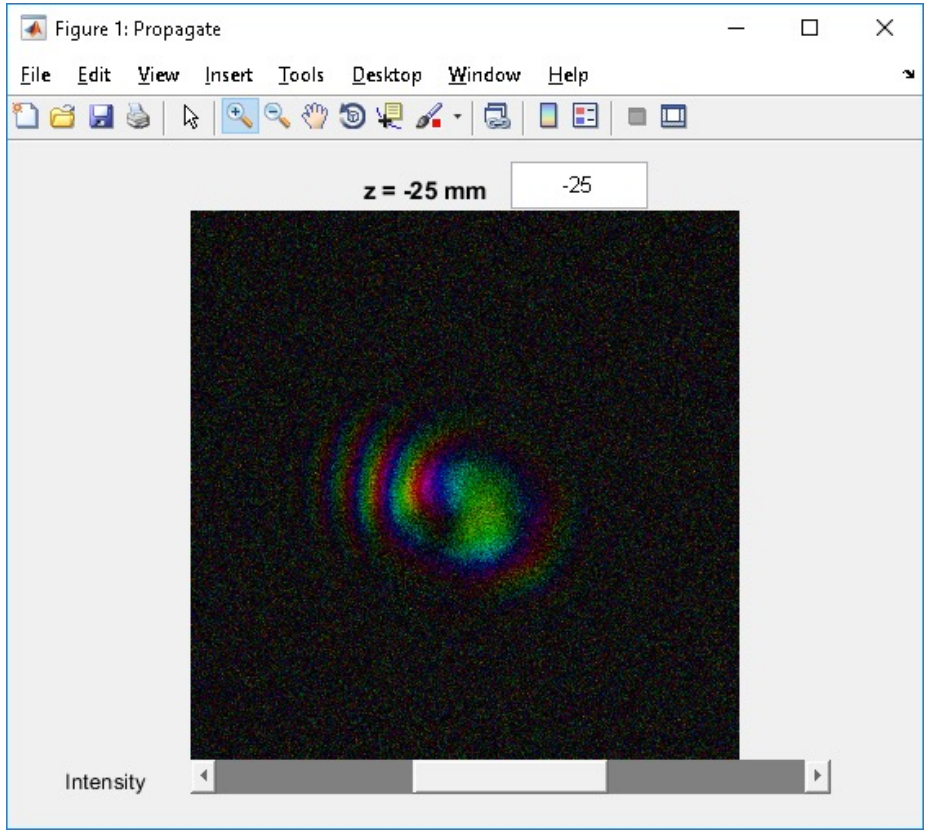
- Need pixel size, distance, error
- Neither M^2 or Gerchberg-Saxton phase retrieval requires information about the lens



Propagate through space!

- Compare with measured beams or propagate through space





Acknowledgements

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