



M^2

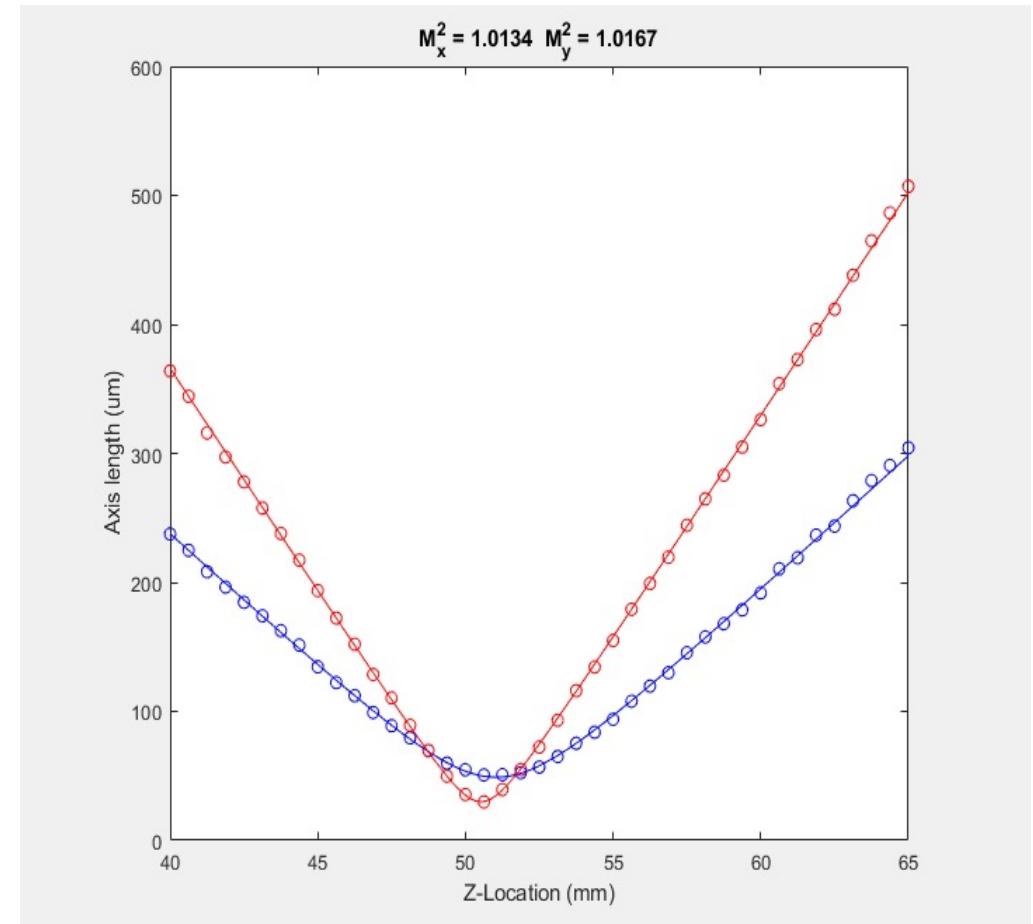
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Group Meeting 8/17/18



What is an M^2 ?

- M^2 is a measure of beam quality/astigmatism
- The M^2 factor indicates how well a beam can be focused.
- M^2 is sometimes called a times diffraction-limited factor
- A perfect diffraction-limited beam has an M^2 of 1 (for a Gaussian beam)



GUI

The screenshot displays the M² GUI interface, which is organized into several functional panels:

- Initialize Objects:** Contains a "Camera Index" input field with the value 2, a "Port (Stage)" dropdown menu set to COM4, and an "Initialize Stage" button.
- Camera Settings:** Includes a "Pixel Size" input field (2.2) and an "Exposure Time" spinner (100). It also features a "Set Region of Interest" sub-panel with "Reset", "X" (0), "X end" (2560), "Y" (0), and "Y end" (1920) inputs, and a "LIVE" button.
- Stage Control:** Contains a "Home Stage" button, a "Current Position (mm)" display showing 0.000, and navigation buttons (< and >), along with an "Update Position" button.
- M² Settings:** Includes input fields for "Wavelength (nm)" (800), "Scan Start (mm)" (40), "Scan Stop (mm)" (65), "# of Steps" (40), and "Step Size (mm)" (0.625). It has "Start Scan!" and "Cancel" buttons.
- Analyze Data from File:** Contains "Compute M²" and "Phase Retrieval" buttons.
- Error message:** A text area for displaying error messages.
- Image Preview:** A 2D plot showing a single bright spot at approximately (1500, 1000) on a dark background. The x-axis is labeled "M² Progress!" and ranges from 0 to 2500. The y-axis ranges from 0 to 1800.
- M² Progress!:** A progress bar at the bottom of the image preview area, currently showing a small blue segment.

Calculating beam properties

- First-Order Moments

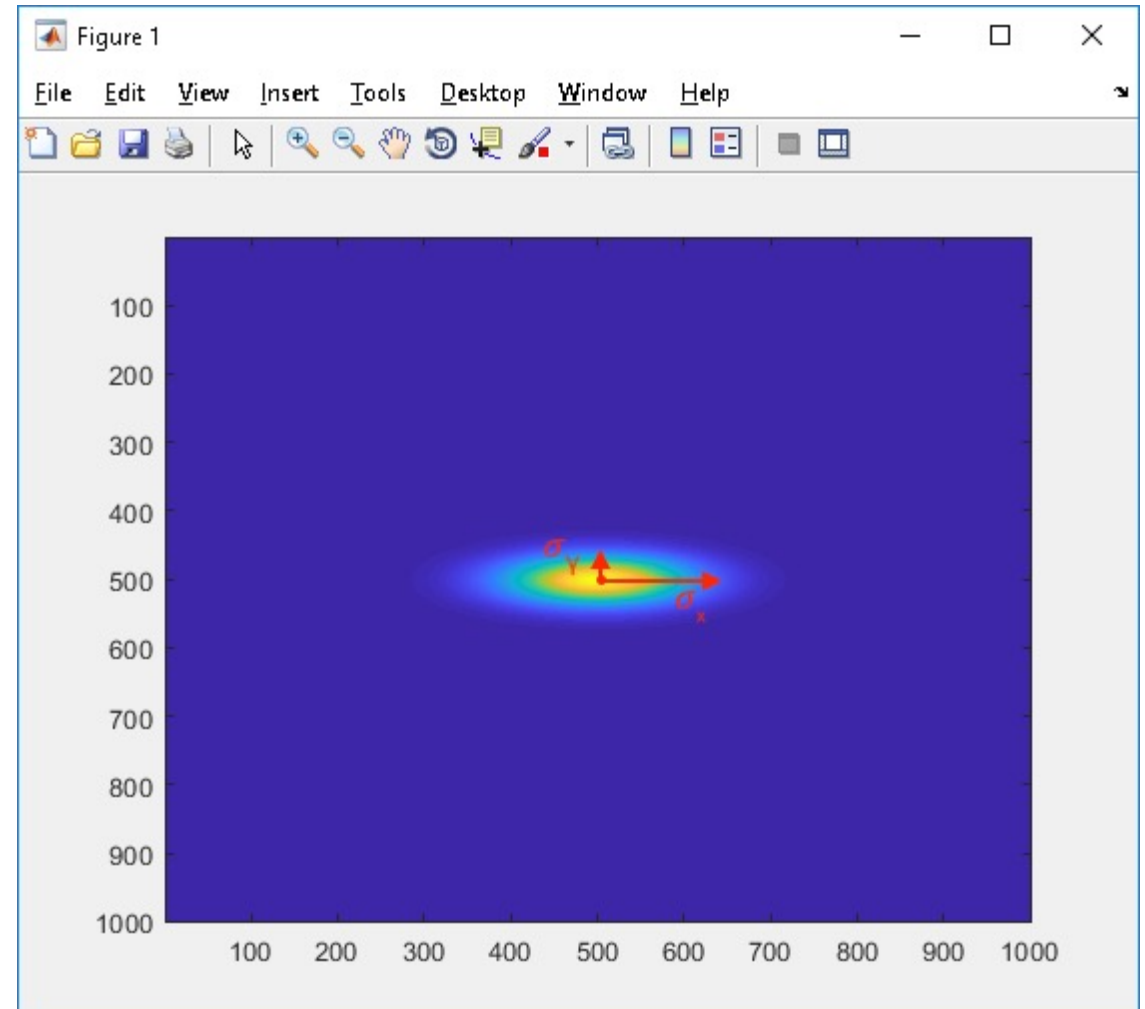
$$\bar{x} = \frac{\iint I(x, y)x \, dx \, dy}{\iint I(x, y) \, dx \, dy} \quad \bar{y} = \frac{\iint I(x, y)y \, dx \, dy}{\iint I(x, y) \, dx \, dy}$$

- Second-Order Moments

$$\sigma_x^2 = \frac{\iint I(x, y)(x - \bar{x})^2 \, dx \, dy}{\iint I(x, y) \, dx \, dy} \quad \sigma_x = HW \frac{1}{\sqrt{e}}$$

$$\sigma_y^2 = \frac{\iint I(x, y)(y - \bar{y})^2 \, dx \, dy}{\iint I(x, y) \, dx \, dy}$$

$$\sigma_{xy}^2 = \frac{\iint I(x, y)(x - \bar{x})(y - \bar{y}) \, dx \, dy}{\iint I(x, y) \, dx \, dy}$$



Calculating beam properties

- Beam widths and the principle axes of the beam can be calculated from the second-order moments

Beam widths:

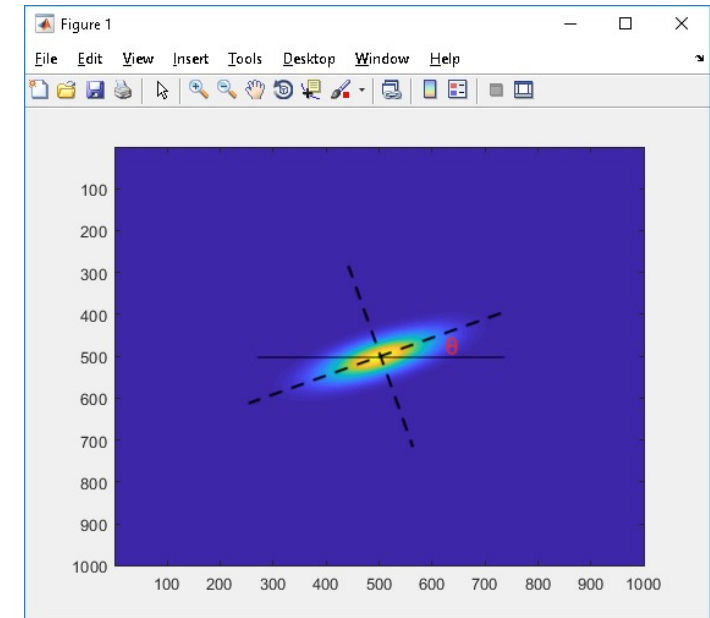
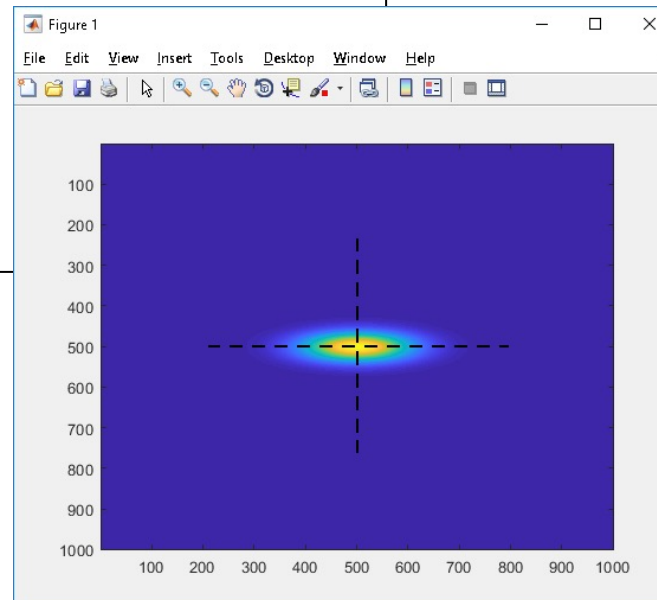
$$d_{\sigma_x}(z) = 2\sqrt{2} \left\{ \left(\sigma_x^2 + \sigma_y^2 \right) + \gamma \left[\left(\sigma_x^2 - \sigma_y^2 \right)^2 + 4 \left(\sigma_{xy}^2 \right)^2 \right]^{\frac{1}{2}} \right\}^{\frac{1}{2}}$$

$$d_{\sigma_x} = FW \frac{1}{e^2}$$

$$d_{\sigma_y}(z) = 2\sqrt{2} \left\{ \left(\sigma_x^2 + \sigma_y^2 \right) - \gamma \left[\left(\sigma_x^2 - \sigma_y^2 \right)^2 + 4 \left(\sigma_{xy}^2 \right)^2 \right]^{\frac{1}{2}} \right\}^{\frac{1}{2}}$$

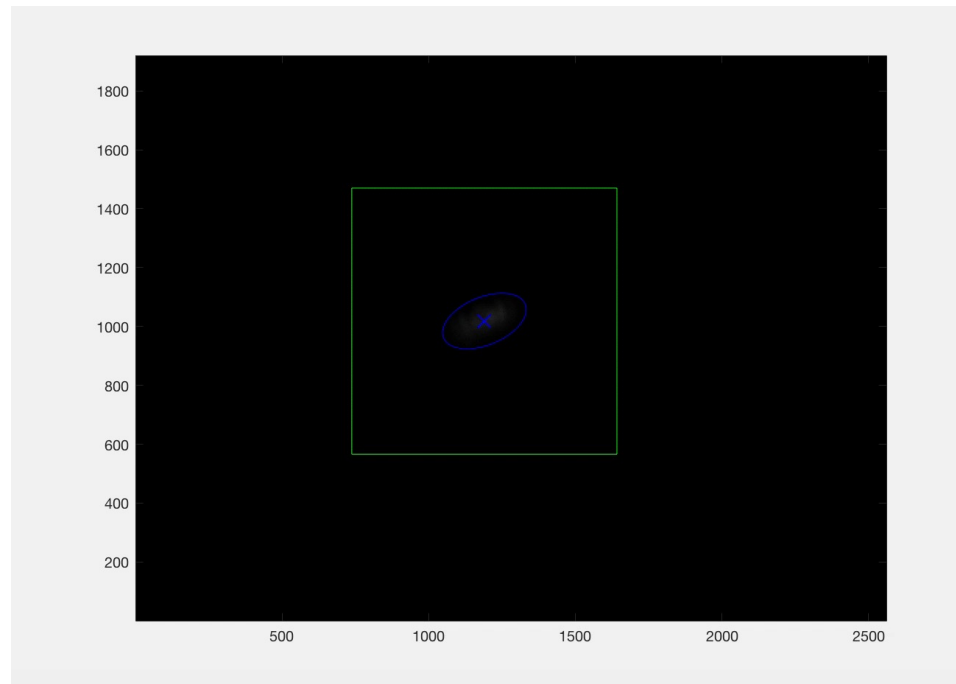
Angle:

$$\varphi(z) = \frac{1}{2} \arctan \left(\frac{2\sigma_{xy}^2}{\sigma_x^2 - \sigma_y^2} \right) \quad \theta = -\frac{\pi}{4} < \theta < \frac{\pi}{4}$$



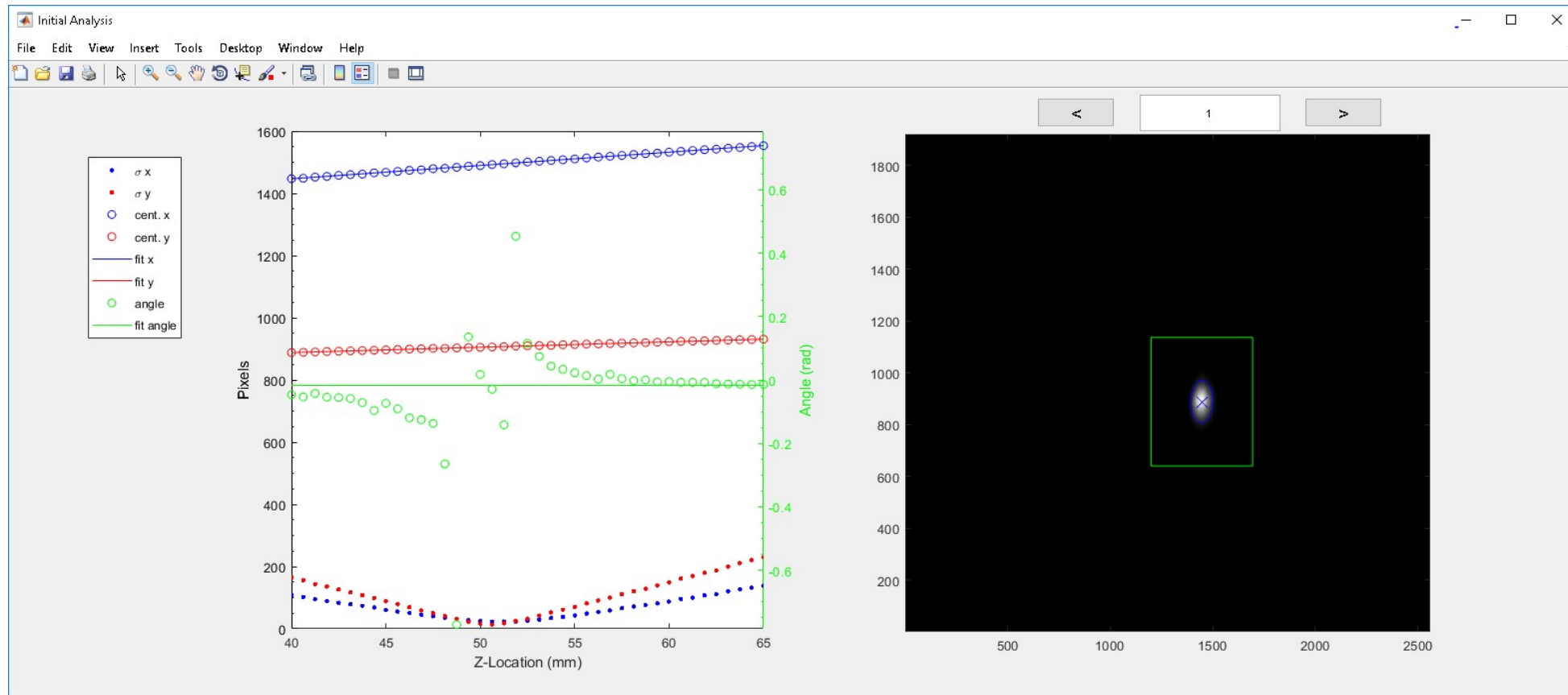
Finding a Region of Interest

- Calculation of σ_x^2 is sensitive to noise on edges
- Also, don't want to crop off beam intensity
- Iteratively find an integration box with size equal to $3d_{\sigma x}$ (or $3d_{\sigma y}$ depending on which is larger)
- Repeat for each image



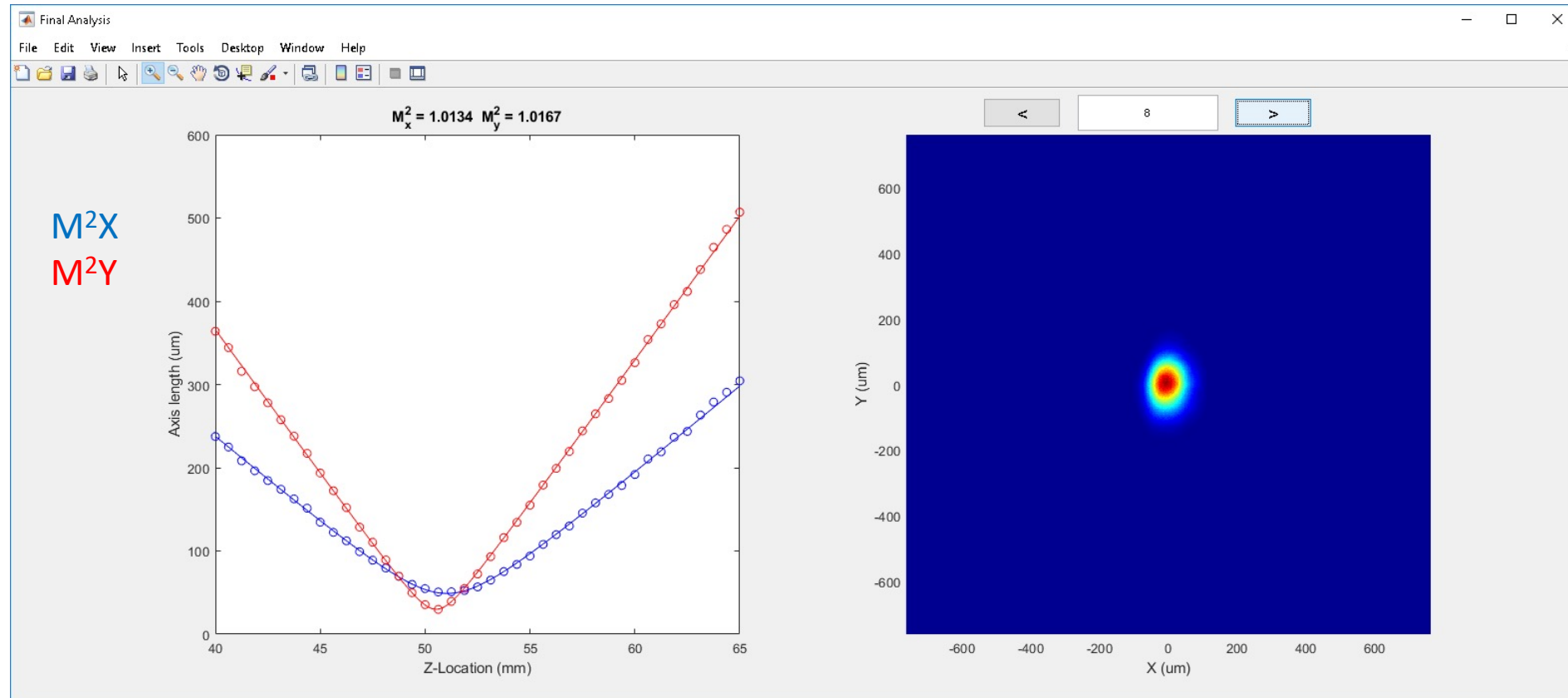
Initial Analysis

- Fit line to angle, because angle becomes unstable around beam waist, thus changing the definitions of d_{σ_x} and d_{σ_y} slightly for each image
- Fit line to centroids to account for possible error



Final Analysis

- Reanalyze images with fitted centroids and angle



Calculating M^2

- ISO Standard Equation returns complex values for some values of a , b , and c

$$d_{\sigma}(z) = \sqrt{a + bz + cz^2}$$

- Instead we used this equation, then converted to ISO Standard definitions of a , b , and c

$$w_R(z) = w_{0R} \left[1 + \left(\frac{z\lambda M^2}{\pi w_{0R}^2} \right)^2 \right]^{1/2}$$

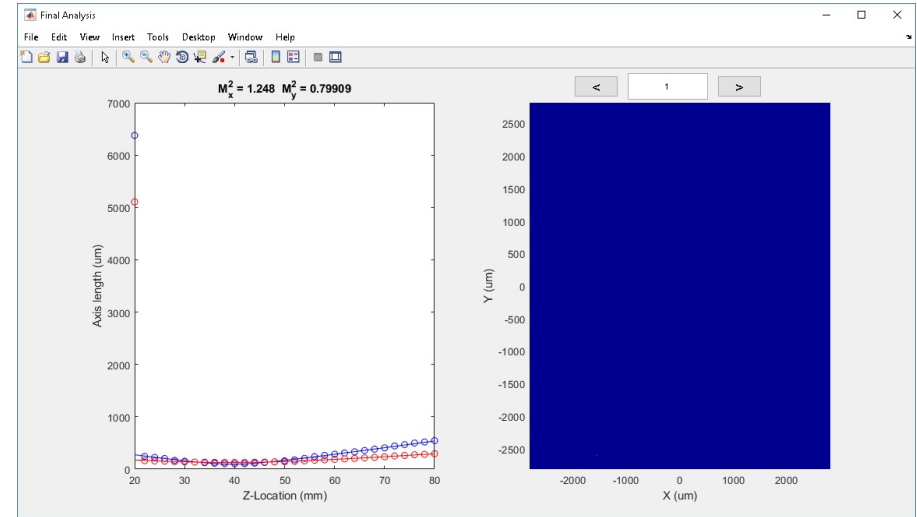
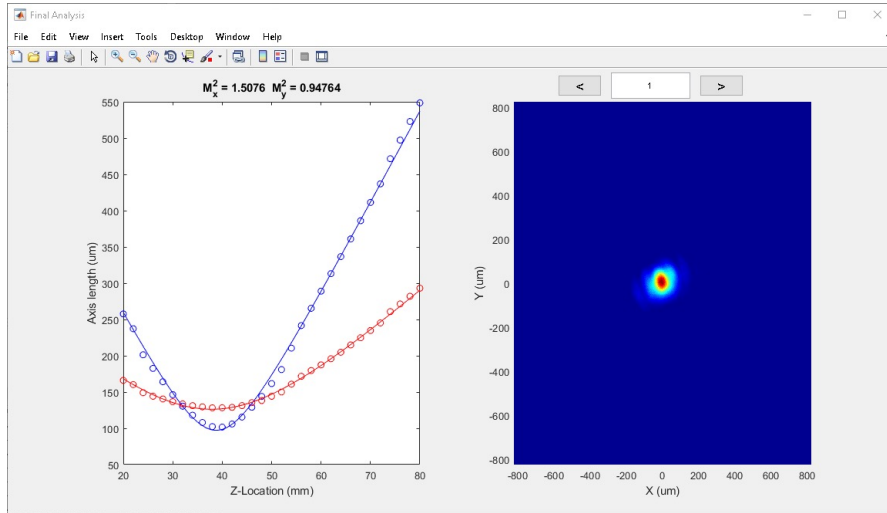
Fit to:

$$f(x) = \sqrt{a^2 + b^2(x - c)^2}$$

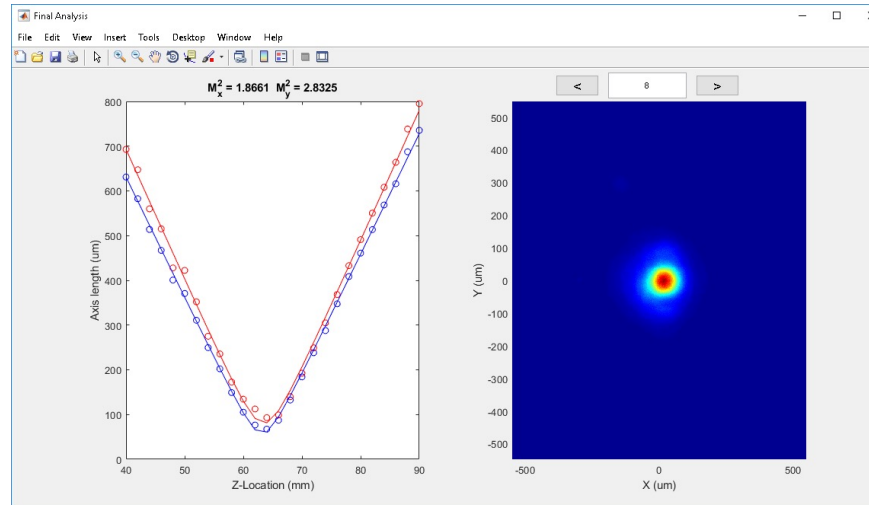
$$M^2 = \frac{\pi}{8\lambda} \sqrt{4ac - b^2}$$

Sample Data

Diode Laser

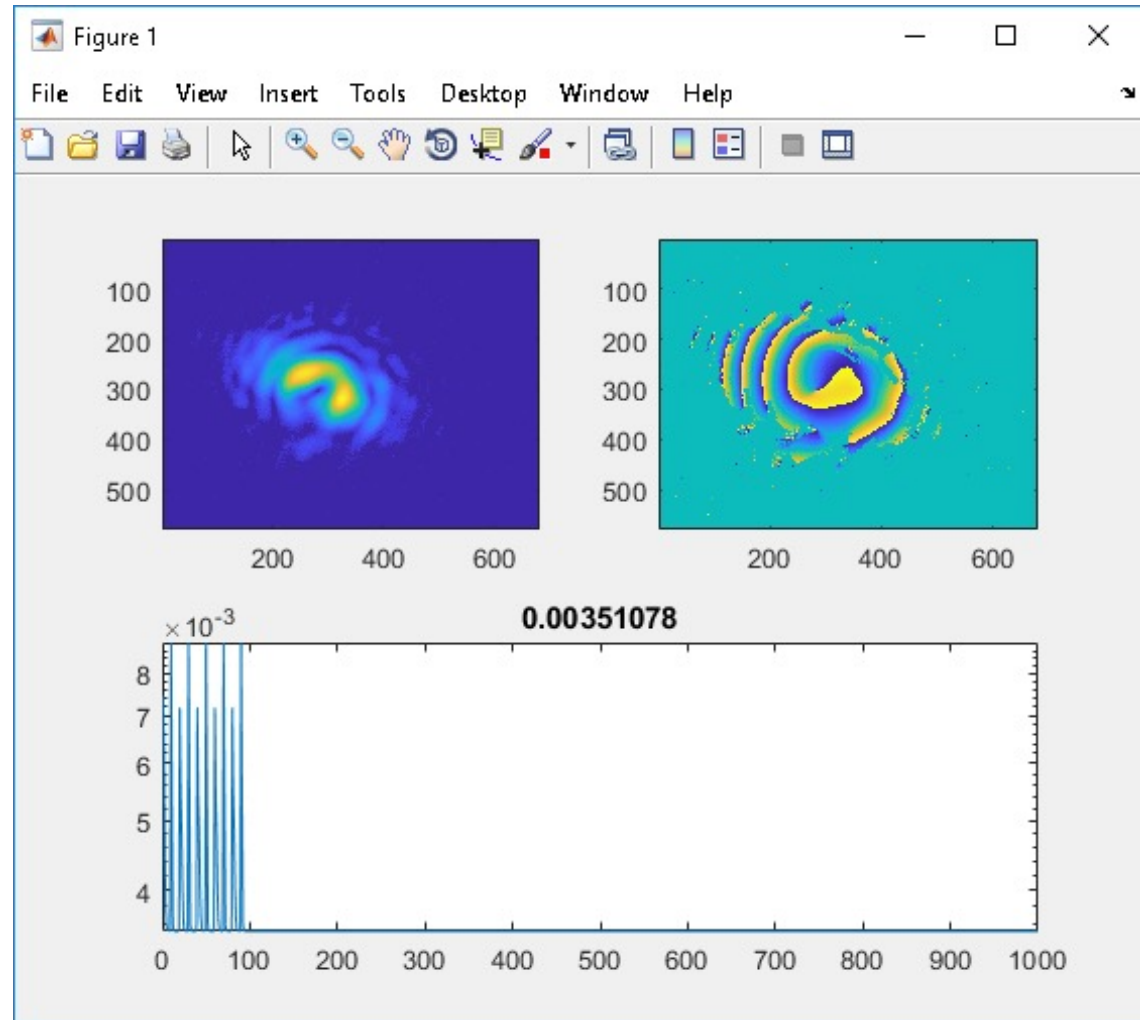


C-Wing Oscillator



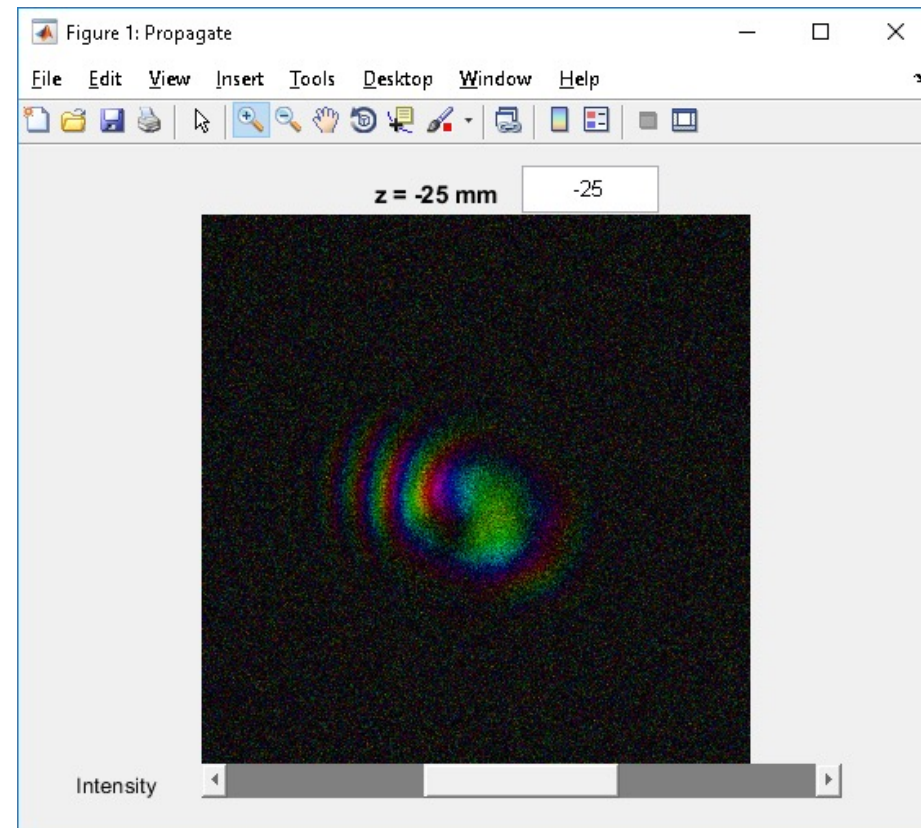
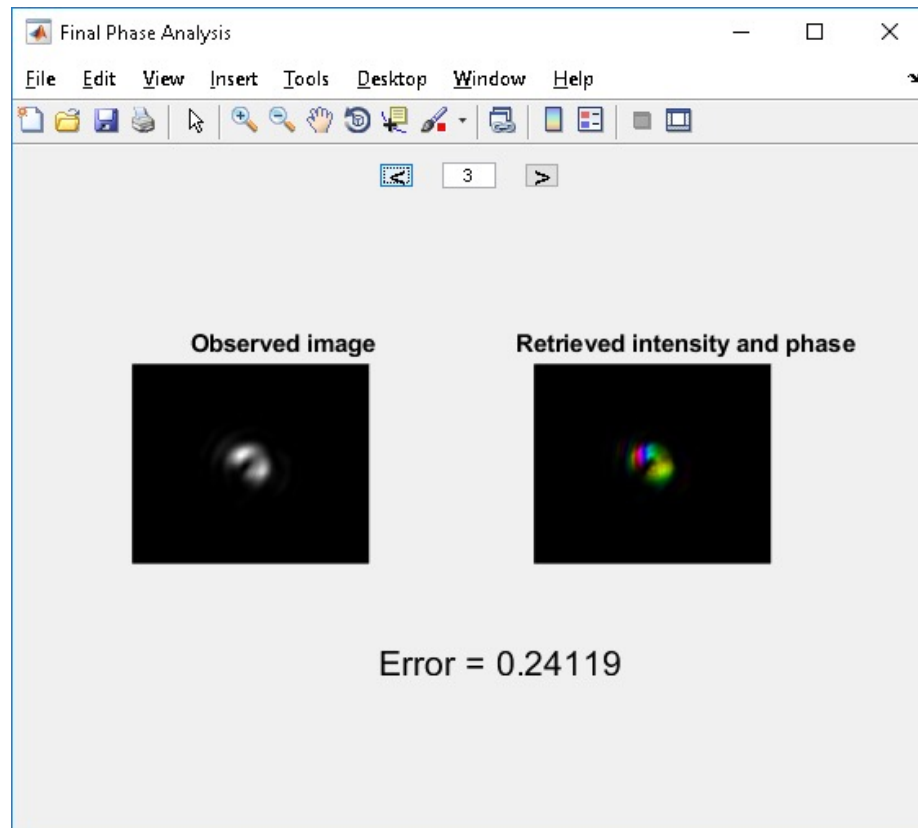
Phase Retrieval using Gerchberg-Saxton

- Need pixel size, distance, error
- Neither M^2 or Gerchberg-Saxton phase retrieval requires information about the lens



Propagate through space!

- Compare with measured beams or propagate through space



Acknowledgements

- David Couch
- Bill Peters
- Michael Tanksalvala
- Kate Uchida
- Kevin Dorney
- Drew Morrill
- Paul Adelgren